Root locus

Steps to solve problems on Root locus

Step. (1) From $G(s)H(s)$, find out no. of open loop poles, zeros & no. of branches

(a) $P > Z$, $N = P$

(b) $Z > P$, $N = Z$

(c) $P = Z$, $N = P = Z$

$P-Z \,/\, Z-P$ branches approaches to ∞.

(2) Draw pole-zero plot. Identify sections of real axis for RL & NRL

Predict for no. of ~~branches~~ breakaway points.

RL → If no. of open loop poles & open loop zeros to R.H.S is ODD

(3) Calculate angle of asymptotes.

No. of asymptotes $= P-Z \,/\, Z-P$.

$$\theta = \frac{(2q+1)180^\circ}{P-Z}, \quad q = 0, 1, 2, \ldots$$

(4) Calculate centroid

$$\sigma = \frac{\Sigma \text{ R.P. of poles} - \Sigma \text{ R.P. zeros}}{P-Z}$$

Sketch for centroid & asymptotes

⑤ Calculate breakaway / breakin points

(i) Find $1 + G(s)H(s) = 0$

(ii) $k = f(s)$ [seperate terms of k & s]

(iii) $\frac{dk}{ds} = 0$

(iv) Find roots of eqⁿ. $\frac{dk}{ds}$ ⇒ breakaway point

(v) Check for their validity

⇓

Substitute value in eqⁿ. $k = f(s)$

If k is +ve ⇒ Valid

k is -ve ⇒ Invalid

⑥ Calculate intersection with Imaginary axis

(i) Find $1 + G(s)H(s) = 0$

(ii) Construct Routh array

(iii) Determine K_{max}

⇓

Value of k for with one of the row of Routh array becomes zero except s^0

(iv) Construct auxiliary eqⁿ $A(s) = 0$ by using coefficients of row which is just above row of zero.

(v) Roots of $A(s)$ for $k = k_{max}$ are intersection points

(7) Calculate angle of departure or arrivals if applicable

[Use only if poles/zeros are complex conjugate]

$$\phi_d = 180° - \phi \quad \text{where } \phi = \Sigma\phi_p - \Sigma\phi_z$$

$$\Sigma\phi_p = \phi_{p1} + \phi_{p2} + ----$$

$$\Sigma\phi_z = \phi_{z1} + \phi_{z2} + ----$$

$$\phi_a = 180° + \phi$$

(8) Combine steps 1 to 7 & draw final sketch of root locus.

(9) Predict for stability

for $0 < k < k_{max}$ ⇒ stable system

for $k = k_{max}$ ⇒ marginally stable

for $k_{max} < k < \infty$ ⇒ unstable stable